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REMARKS ON DIVISION.

By J. K. ELLWOOD, A. M., Principal of Colfax School, Pittsburg, Pennsylvania.

A large majority of the arithmetics in use in the United States teach that a "concrete number" can be divided by an abstract or pure number; that if the dividend is \$10 and the divisor 2, the quotient is \$5. Why this has been allowed to go unchallenged for so many generations is a psychological rather than a mathematical problem. Mathematicians have been neither scarce nor idle, but they seem to have been working upward and outward among the branches instead of digging down to the rootlets of the infinite tree of mathematical truth.

In the realm of number all the human mind can do is unite and take apart, involve and evolve, compose and analyze. Everthing is based upon addition. The inverse of additon is subtraction. Multiplication is a mere process of adding, hence *its* inverse is subtraction. If a given product is \$20 and the multiplicand \$5, we can find the multiplier by subtracting \$5 from \$20 until nothing remains. The number of times we subtract is 4, the multiplier. One number can be taken from another just as often as it is contained therein; hence, division is equivalent to subtraction, and is the inverse of multiplication.

If a given product is \$20, and the multiplier 4, we cannot by mere subtraction find the multiplicand. That is to say, multiplication has but one in-

verse, which is subtraction or its equivalent, division. But by subtraction we can find only *how many times*; hence, by division the only thing that can be found is *how many times*. That is, how many times one number is contained in another of the same kind. It is thus seen that quotient is always *abstract*. Consequently, the dividend and divisor must be like numbers; for if a quotient is a, the dividend is a times the divisor, whatever it may be. Therefore, a "concrete number" can not be divided by an abstract one.

To find one of the equal parts of a concrete number is more than division: it is a problem that involves the use of division; it is an "application" of division, just as "profit and loss" is an application of percentage. Thus, to find $\frac{1}{4}$ of \$20, we proceed logically as follows:

(*a*) $\frac{1}{4}$ of \$20 is as many dollars as there are 4's in 20.

(*b*) There are *five* 4's in 20.

(*c*) $\therefore$ $\frac{1}{4}$ of \$20 is \$5.

The reasoning in such problems must be in the abstract, and the result interpreted or applied in the conclusion. But pure division involves none of this reasoning—it involves only a retracing of the steps in multiplication or addition.

It is plain that to find $\frac{1}{4}$ of a number is to divide that number by 4. To find $\frac{1}{4}$ of $\frac{8}{9}$ is to divide $\frac{8}{9}$ by 4. Hence, the alleged "compound fraction" is no fraction at all; it is not even an example in multiplication of fractions, as given by all arithmetics, but it is clearly an example in *division* of fractions.

Expressions like $\frac{4}{\frac{2}{3}}$, commonly called "complex fractions", are not fractions; they are indicated *divisions*. They have the *form* of a fraction, but so has an Indian tobacco sign the form of a man. Unexecuted division and ratio may be expressed in fractional form, but a fraction expresses neither division nor ratio. Thus, $8\div 9$ may be written $\frac{8}{9}$; but this does not denote 8 of the nine equal parts of "a unit." When $\frac{8}{9}$ expresses a division to be performed, the 9 is a number—*nine;* when it is a fraction, the 9 is a name—*ninths*.

In the former case the expression is read 8 divided by nine; in the latter it is read 8 ninths.

Besides, if $\frac{4}{\frac{2}{3}}$ were a fraction, the denominator $\frac{2}{3}$ would indicate that some unit had been divided into $\frac{2}{3}$ equal parts! Since it is impossible for *man* to so divide a unit, this species of complex fractions must be regarded as a special gift from on high, "for with God all things are possible."

ARITHMETIC.

Conducted by B. F. FINKEL, Kidder, Missouri. All contributions to this department should be sent to him

5. **Proposed by E. E. KINNEY, Anaconda, Montana.**

A board is 16 inches long and 9 inches wide. How may it be cut in two